RECOGNISING ACHIEVEMENT

ADVANCED GCE UNIT

MATHEMATICS

4730/01

Mechanics 3

WEDNESDAY 10 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

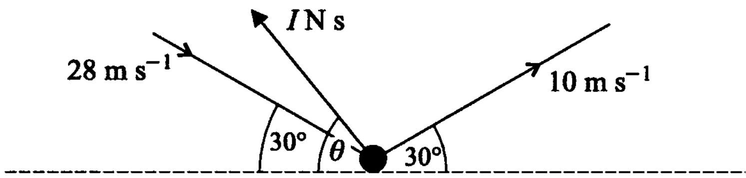
ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

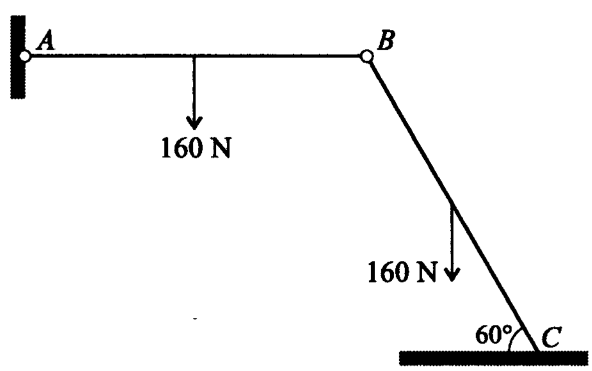
1 A particle P of mass 0.6 kg is attached to a fixed point O by a light inextensible string of length 0.4 m. While hanging at a distance 0.4 m vertically below O, P is projected horizontally with speed 5 m s^{-1} and moves in a complete vertical circle. Calculate the tension in the string when P is vertically above O. [6]

2



When a tennis ball of mass 0.057 kg bounces it receives an impulse of magnitude I N s at an angle of θ to the horizontal. Immediately before the ball bounces it has speed 28 m s^{-1} in a direction of 30° to the horizontal. Immediately after the ball bounces it has speed 10 m s^{-1} in a direction of 30° to the horizontal (see diagram). Find I and θ. [7]

3



Two identical uniform rods, AB and BC, are freely jointed to each other at B, and A is freely jointed to a fixed point. The rods are in limiting equilibrium in a vertical plane, with C resting on a rough horizontal surface. AB is horizontal, and BC is inclined at 60° to the horizontal. The weight of each rod is 160 N (see diagram).

(i) By taking moments for AB about A, find the vertical component of the force on AB at B. Hence or otherwise find the magnitude of the vertical component of the contact force on BC at C. [3]

(ii) Calculate the magnitude of the frictional force on BC at C and state its direction. [4]

(iii) Calculate the value of the coefficient of friction at C. [2]

4 A particle P of mass 0.2 kg is suspended from a fixed point O by a light elastic string of natural length 0.7 m and modulus of elasticity 3.5 N. P is at the equilibrium position when it is projected vertically downwards with speed 1.6 m s^{-1}. At time t s after being set in motion P is x m below the equilibrium position and has velocity $v\text{ m s}^{-1}$.

(i) Show that the equilibrium position of P is 1.092 m below O. [3]

(ii) Prove that P moves with simple harmonic motion, and calculate the amplitude. [5]

(iii) Calculate x and v when $t = 0.4$. [5]

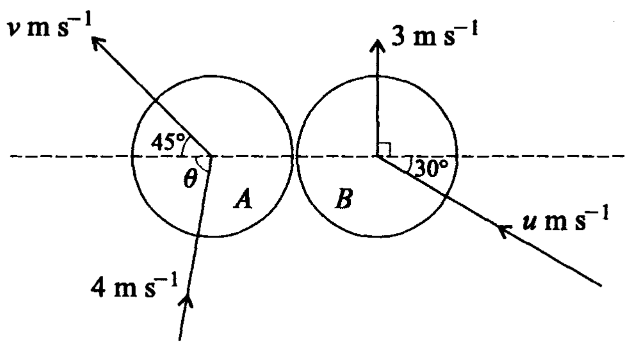
5 The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass m kg and is released from rest. When a grain has fallen a distance x m, it has speed $v\,\text{m s}^{-1}$. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude mkv^2 N, where k is a positive constant.

(i) Show that $\left(\frac{v}{g - kv^2}\right)\frac{dv}{dx} = 1$. [2]

(ii) Find v^2 in terms of k, g and x. Hence show that, as x becomes large, the limiting value of v is $\sqrt{\frac{g}{k}}$. [7]

(iii) Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find k. [3]

6



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.4 kg, and B has mass m kg. Immediately before the collision, A is moving with speed $4\,\text{m s}^{-1}$ at an acute angle θ to the line of centres, and B is moving with speed $u\,\text{m s}^{-1}$ at 30° to the line of centres. Immediately after the collision A is moving with speed $v\,\text{m s}^{-1}$ at 45° to the line of centres, and B is moving with speed $3\,\text{m s}^{-1}$ perpendicular to the line of centres (see diagram).

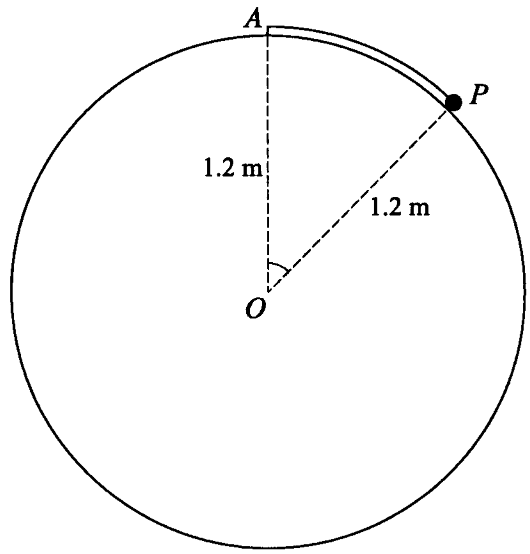
(i) Find u. [2]

(ii) Given that $\theta = 88.1°$ correct to 1 decimal place, calculate the approximate values of v and m. [5]

(iii) The coefficient of restitution is 0.75. Show that the exact value of θ is a root of the equation $8\sin\theta - 6\cos\theta = 9\cos 30°$. [5]

[Question 7 is printed overleaf.]

7



The diagram shows a particle P of mass 0.5 kg attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 1.2 m. The string has natural length 0.6 m and modulus of elasticity 6.86 N. P is released from rest at a point on the surface of the sphere where the acute angle AOP is at least 0.5 radians.

(i) (a) For the case angle $AOP = \alpha$, P remains at rest. Show that $\sin \alpha = 2.8\alpha - 1.4$. [4]

(b) Use the iterative formula

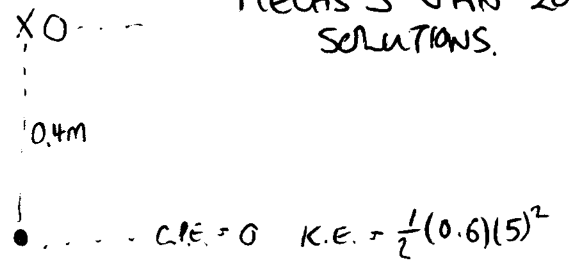
$$\alpha_{n+1} = \frac{\sin \alpha_n}{2.8} + 0.5,$$

with $\alpha_1 = 0.8$, to find α correct to 2 significant figures. [2]

(ii) Given instead that angle $AOP = 0.5$ radians when P is released, find the speed of P when angle $AOP = 0.8$ radians, given that P is at all times in contact with the surface of the sphere. State whether the speed of P is increasing or decreasing when angle $AOP = 0.8$ radians. [7]

MECHS 3 JAN 2007
SOLUTIONS.

1.



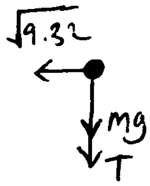
CONSERVATION OF ENERGY

K.E. @ BOTTOM = { G.P.E. + K.E. } AT TOP

$(0.5)(0.6)(5)^2 = 0.6g \times 0.8 + \frac{1}{2} \times 0.6 \times v^2$

$\therefore \quad v^2 = 9.32$

@ TOP.

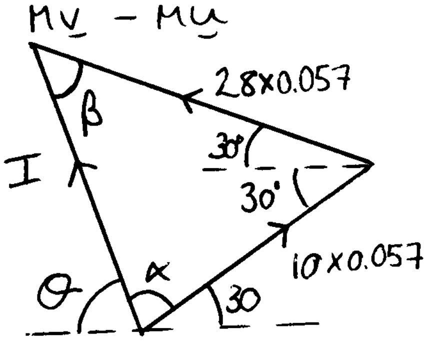


$mg + T = \frac{mv^2}{r}$

$T = 0.6\left(\frac{9.32}{0.4} - 9.8\right) = \underline{\underline{8.1\ [N]}}$

2.

$\underline{I} = M\underline{v} - M\underline{u}$



Cosine Rule

$I^2 = (10^2 + 28^2 - 2(10)(28)\cos 60)\, m^2$

$I^2 = 604 m^2 \Rightarrow I = 24.5764 1145\, m = 24.5764 \times 0.057$

$I = 1.40 \ [Ns] \ (3s.f.)$

- Sine Rule

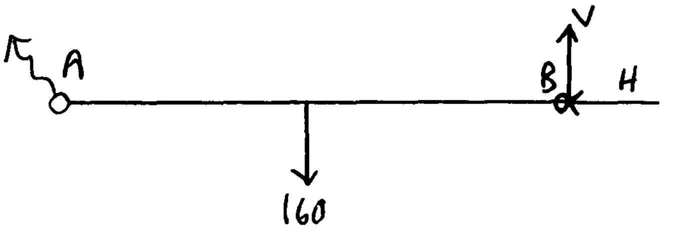
$\sin\alpha = \frac{0.057 \times 28 \sin 60}{1.40\ldots} \Rightarrow \alpha = \sin^{-1}(0.98666\ldots) = 80.633\ldots$ or $180 - 80.633 = 99.367\ldots$

$\therefore \quad \theta = 180 - \cancel{80.633}\ 99.367 - 30$

$\underline{\underline{\theta = \cancel{69.4^\circ}}}\ \underline{\underline{50.6^\circ}}$

need to check by calculating β
(β = 20.646 ∴ α = 99.367)

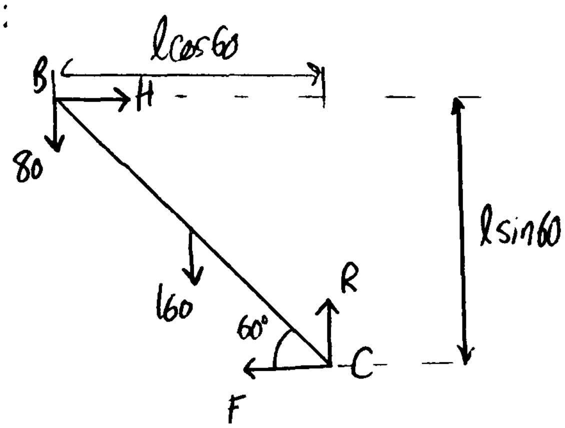
3. (i) Consider AB:



$M(A) \circlearrowleft^{+}$ $\quad V\ell - 160\frac{\ell}{2} = 0$

$$\underline{\underline{V = 80\ [N]}}$$

Consider BC:



$R(\uparrow)$ $\quad R = 80 + 160 = \underline{\underline{240\ [N]}}$

(ii) $M(B)$ for BC: $\circlearrowleft^{+}$

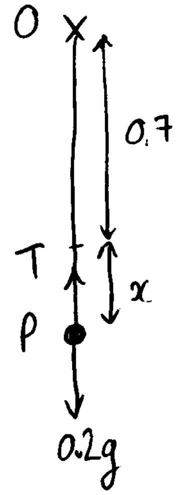
$$R\ell\cos 60 - 160\frac{\ell}{2}\cos 60 - F\ell\sin 60 = 0$$

$$F = \frac{(120 - 40)2}{\sqrt{3}} = \frac{160\sqrt{3}}{3} = \underline{\underline{92.4\ [N]}}\ (3\text{s.f.}) \text{ in direction } \overrightarrow{BA}.$$

(iii) Limiting Equilibrium $\therefore$ $F = F_{max} = \mu R$

$$\therefore \quad \mu = \frac{92.36\ldots}{240} = \underline{\underline{0.385}}\ \ (3\text{s.f.})$$

4. (i) Consider equilibrium



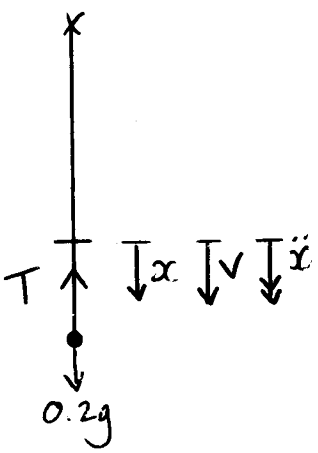
R(↑) $T = 0.2g$

$\therefore \frac{\lambda x}{l} = 0.2g$

$\therefore x = \frac{0.2 \times 9.8 \times 0.7}{3.5} = 0.392$

$\therefore OP = 0.7 + 0.392 = \underline{\underline{1.092 \text{ [m]}}}$

(ii) Consider general position:



N(II) ↓

$0.2g - \frac{\lambda x}{l} = m\ddot{x}$ (×5) $\quad m = 0.2$

$\therefore \ddot{x} = 9.8 - 25x = -25\left(x - \frac{9.8}{25}\right)$

let $x - \frac{9.8}{25} = z$

Then $\dot{x} = \dot{z}$ and $\ddot{x} = \ddot{z}$

$\therefore \ddot{z} = -25z$ SHM. WITH PERIOD $\underline{\underline{\frac{2\pi}{5}}}$

about $z = 0$ $(x = 0.392)$

When $z = 0$, $v = 1.6$, $n = 5$, $t = 0$,.

$v^2 = n^2(a^2 - z^2)$

$\therefore a = \sqrt{\frac{v^2}{n^2} + z^2} = \sqrt{\frac{1.6^2}{25}} = \frac{1}{5}(1.6) = \underline{\underline{0.32 \text{ [m]}}}$

(iii) Since @ $t = 0$ $z = 0$ $\quad z = a\sin(nt)$

So $x = a\sin(nt) + 0.392$

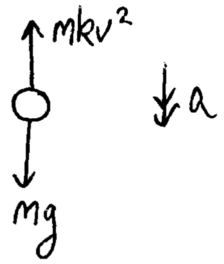
When $t = 0.4$ $\quad x = 0.32\sin(0.4 \times 5) + 0.392 = \underline{\underline{0.683 \text{ [m]}}}$ (3sf)

i.e. 0.291 [m] below equilibrium position.

& $v = an\cos(nt) = 0.32 \times 5\cos(0.4 \times 5)$

$= \underline{\underline{-0.666 \text{ [ms}^{-1}\text{]}}}$ (↑ upwards)

5.(i) CONSIDER A GRAIN OF SAND:



$a = \frac{dv}{dt} = v\frac{dv}{dx}$

N(II) ↓ $\quad mg - mkv^2 = mv\frac{dv}{dx}$

$\div m(g-kv^2)$

$\left(\frac{v}{g-kv^2}\right)\frac{dv}{dx} = 1$ as required.

(ii)

$$\int_0^V \frac{v}{g-kv^2}\frac{dv}{dx}\,dx = \int_0^x 1\,dx$$

$$\left[-\frac{1}{2k}\ln\left|g-kv^2\right|\right]_0^V = x$$

$$-\frac{1}{2k}\ln\left|g-kV^2\right| - -\frac{1}{2k}\ln\left|g\right| = x \qquad \times 2k$$

$$\ln\left|\frac{g}{g-kV^2}\right| = 2kx$$

$$\frac{g}{g-kV^2} = e^{2kx}$$

$$g = e^{2kx}(g-kV^2)$$

$$\therefore V^2 = \frac{g(e^{2kx}-1)}{ke^{2kx}} \qquad \times \frac{e^{-2kx}}{e^{-2kx}}$$

$$V^2 = \underline{\underline{\frac{g(1-e^{-2kx})}{k}}}$$

as $x \to \infty$, $e^{-2kx} \to 0$ $\quad V^2 \to \frac{g}{k}$

$\therefore \quad V \to \underline{\underline{\sqrt{\frac{g}{k}}}}$

5(iii)

$x = 300$, $v = 0.9\sqrt{\frac{g}{k}}$

$0.81\frac{\cancel{g}}{\cancel{k}} = \frac{\cancel{g}}{\cancel{k}}\left(1 - e^{-2k(300)}\right)$

$e^{-600k} = 1 - 0.81 = 0.19$

$-600k = \ln 0.19$

$k = -\frac{1}{600}\ln 0.19$

$= \underline{\underline{0.00277}}$

6. (i) $u\sin 30 = 3$

$\underline{\underline{u = 6\,[ms^{-1}]}}$

(ii) $4\sin 88.1° = v\sin 45°$

$v = 4\sqrt{2}\sin 88.1° = 5.653744...$

$= \underline{\underline{5.65\,[ms^{-1}]}}$ (3sf.)

$0.4(4\cos 88.1) - 6m\cos 30 = -0.4 \times 5.6537...\cos 45 + 0.$

$m = \frac{0.4(4\cos 88.1 + 5.6537\cos 45)}{6\cos 30}$

$= 0.318\,[kg]$ (3sf.)

(iii)

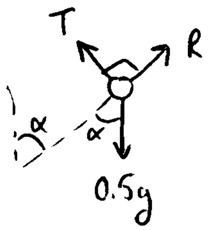
$e = \frac{V_{sep}}{V_{app}}$ (HORIZONTALLY)

$0.75 = \frac{v\cos 45}{4\cos\theta + u\cos 30}$ and $v = \frac{4\sin\theta}{\sin 45} = 4\sqrt{2}\sin\theta.$

$\therefore \frac{3}{4} = \frac{4\sin\theta}{4\cos\theta + 6\cos 30} \Rightarrow \underline{\underline{8\sin\theta - 6\cos\theta = 9\cos 30°}}$ QED.

(Rearrange easily.)

7.(i) CONSIDER EQUILIBRIUM



R($\nwarrow$) $T = 0.5g \sin\alpha$ {1}

Now $T = \dfrac{\lambda x}{l}$ and $(l + x) = r\alpha$ (length of arc $= r\theta$)

so $x = r\alpha - l$

$\therefore \quad T = \dfrac{\lambda(r\alpha - l)}{l} = \dfrac{6.86(1.2\alpha - 0.6)}{0.6}$ {2}

$\therefore \quad 6.86(1.2\alpha - 0.6) = 0.6(0.5g\sin\alpha)$

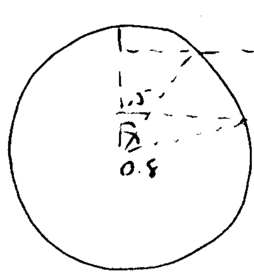
...

$\sin\alpha = 2.8\alpha - 1.4$ QED

(ii)

$$\left.\begin{aligned} \alpha_1 &= 0.8 \\ \alpha_2 &= 0.7561 \\ \alpha_3 &= 0.7450 \\ \alpha_4 &= 0.7421 \\ \alpha_5 &= 0.7413 \\ \alpha_6 &= 0.7411 \\ \alpha_7 &= 0.7411 \ldots \end{aligned}\right\} \quad \alpha = 0.74 \text{ (2s.f)}$$

(iii) smooth sphere, so energy generally conserved.



K.E. = 0 G.P.E = 0 E.P.E = $\dfrac{\lambda x^2}{2l} = \dfrac{6.86(1.2\times0.5 - 0.6)^2}{2(0.6)} = 0$

G.P.E = $-0.5g\times(1.2\cos0.5 - 1.2\cos0.8)$, K.E. $= \frac{1}{2}\times0.5v^2 = \frac{1}{4}v^2$

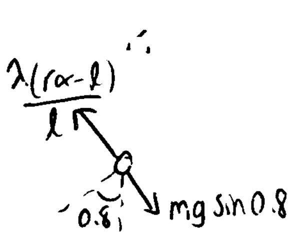
E.P.E = $\dfrac{6.86(1.2\times0.8 - 0.6)^2}{2(0.6)}$

Since equilibrium @ $\alpha = 0.74$ particle will be decellerating @ $\alpha = 0.8$.
(See Tension calc for confirmation, below)

$\therefore$

$v^2 = 4\left(0.5\times9.8(1.2\cos0.5 - 1.2\cos0.8) - \dfrac{6.86}{1.2}(1.2\times0.8 - 0.6)^2\right)$

$v = 1.14\ ms^{-1}$



$\left.\begin{aligned} & mg\sin0.8 = 3.515\ldots \\ & T = \dfrac{6.86(1.2\times0.8 - 0.6)}{0.6} = 4.116\ldots \end{aligned}\right\}$ $\therefore$ Speed is decreasing (a is -ve).